

Fig. 1. One asymmetrical unit of $\beta_{1}-\mathrm{K}_{2} \mathrm{UF}_{6}$. One-fourth unit cell is outlined.

Table 3. Observed and calculated structure factors for $\beta_{1}-\mathrm{K}_{2} \mathrm{UF}_{6}, F_{o}, F_{c}$ and $\alpha \times 10$


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The crystal structure of $\boldsymbol{\gamma}-\mathbf{N a}_{2} \mathbf{Z r F}_{6}{ }^{*}$ By George Brunton, Reactor Chemistry Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830, U.S.A.
(Received 25 January 1969)
The crystal structure of $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ is a new structure type with space group $P 2_{1} / c$. The Zr ions are coordinated by $7 \mathrm{~F}^{-}$ions at distances of 2.012 to $2.167 \AA$. The structure is metastable below $460^{\circ} \mathrm{C}$.

The complex fluoride $\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ has four polymorphs (Barton, Grimes, Insley, Moore \& Thoma, 1958) which can be distinguished from one another by their optical and X-ray properties. $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ is biaxial positive; $2 V=75^{\circ}, N_{x}=$ $1 \cdot 408$ and $N_{z}=1 \cdot 412$. The equilibrium $\beta-\gamma$ transition is at

[^0]505C and the $\gamma-\delta$ transition is at 460 C . The crystal structure of $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ is a new structure type.

## Experimental

Single crystals of $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ were obtained from a rapidly cooled ingot of the composition $\mathrm{NaF}-\mathrm{ZrF}_{4}-\mathrm{UF}_{4}$ (55-40-5 mole\%). The crystals were ground in a small air race, and

Table 1. The lattice and atomic parameters of $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$

|  | $x \times 10^{3}$ | $y \times 10^{3}$ | $z \times 10^{3}$ | $\beta_{11}{ }^{*} \times 10^{3}$ | $\beta_{22} \times 10^{3}$ | $\beta_{33} \times 10^{4}$ | $\beta_{12} \times 10^{3}$ | $\beta_{13} \times 10^{4}$ | $\beta_{23} \times 10^{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zr | $176 \cdot 1$ (2) | $448 \cdot 0$ (3) | 98.90 (5) | $21 \cdot 0$ (6) | 18 (2) | $20 \cdot 0$ (7) | $0 \cdot 8$ (3) | 4 (2) | $1 \cdot 0$ (7) |
| $\mathrm{Na}(1)$ | $326 \cdot 7$ (8) | 499 (2) | $407 \cdot 7$ (3) | 26 (3) | 25 (3) | 27 (3) | -2 (2) | 11 (6) | -4 (5) |
| $\mathrm{Na}(2)$ | $246 \cdot 7$ (9) | 962 (2) | $247 \cdot 7$ (4) | 26 (2) | 33 (4) | 34 (3) | 4 (2) | 24 (6) | 15 (6) |
| F(1) | 20 (2) | 709 (2) | $335 \cdot 8$ (5) | 31 (3) | 15 (4) | 33 (3) | 2 (3) | -4 (7) | 2 (8) |
| $\mathrm{F}(2)$ | 55 (2) | 176 (2) | $350 \cdot 3$ (5) | 27 (3) | 35 (4) | 33 (3) | -7 (3) | 14 (6) | 10 (9) |
| F(3) | 346 (2) | 224 (2) | $24 \cdot 2$ (5) | 31 (3) | 22 (4) | 39 (3) | 1 (3) | 5 (7) | 1 (9) |
| F (4) | 598 (2) | 229 (2) | 364.8 (4) | 30 (3) | 28 (4) | 27 (3) | 3 (2) | 11 (6) | 7 (7) |
| F(5) | 430 (2) | 286 (2) | $183 \cdot 5$ (4) | 29 (3) | 37 (5) | 30 (3) | -1 (3) | -11(7) | 31 (9) |
| $F(6)$ | 871 (2) | 328 (2) | $11 \cdot 6$ (4) | 23 (2) | 17 (4) | 32 (3) | -4 (2) | -9(6) | 9 (8) |

* Coefficients in the temperature factor: $\exp \left[-\left(\beta_{11} h^{2}+\beta_{22} k^{2}+\beta_{33} l^{2}+2 \beta_{12} h k+2 \beta_{13} h l+2 \beta_{23} k l\right)\right]$. The number in parentheses is the standard error in terms of the last significant figure.
an ellipsoid $0.156 \times 0.156 \times 0.208 \mathrm{~mm}$ was mounted on a computer operated Picker four-circle goniostat. The intensities of 489 independent reflections were measured with a scintillation-counter detector using unfiltered $\mathrm{Cu} K \alpha$ radiation ( $K \alpha_{1}=1.54050$ and $K \alpha_{2}=1.54434$ ) out to $145^{\circ} 2 \theta$ by the $2 \theta$ scan technique. The linear absorption for $\mathrm{Cu} K \alpha$ radiation by $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ is $225.53 \mathrm{~cm}^{-1}$ and an absorption correction was calculated for each reflection.


## Refinement and results

The observed conditions for diffraction: $h k l-$ no conditions, $h 0 l-l=2 n$ and $0 k 0-k=2 n$ are consistent for the space group $P 2_{1} / c$ (14). The lattice parameters were refined from 17 $\mathrm{Cu} K \alpha_{1}$ reflections in the $2 \theta$ range $92-110^{\circ}$, Table 1.

The structure was refined by iterative least-squares using a modification of the Busing, Martin \& Levy (1962) computer program. The starting parameters were determined from a three-dimensional Patterson function, and the results are listed in Table 1. An extinction correction was made on $F_{c}$ by the method suggested by Zachariasen (1967). The scattering factors for the ions were taken from Cromer \& Waber (1965) and the values of $\Delta f^{\prime}=-0.6$ and $\Delta f^{\prime \prime}=2.5$ electrons were used for the anomalous dispersion of $\mathrm{Cu} K \alpha$ radiation by zirconium (Dauben \& Templeton, 1955).

The quantity minimized by the least-squares program was $\Sigma w\left|\left|s F_{o}\right|-\left|F_{c}\right|\right|^{2}$ with weights, $w$, equal to the reciprocals of the variances which were estimated from the emprirical equation:

$$
\sigma^{2}\left(F_{o}^{2}\right)=\left[T+B+\{0 \cdot 15(T-B)\}^{2}\right] /\left[A(\mathrm{Lp})^{2}\right]
$$

where $T=$ total counts, $B=$ background counts, $A=$ absorp-
tion correction and $\mathrm{Lp}=$ Lorentz-polarization (Brown \& Levy, 1964); $\sigma\left(F_{o}\right)=\sigma\left(F_{o}^{2}\right) / 2 F_{o}$. Anisotropic temperature factors were calculated for all atoms (Table 1) and a stereoscopic pair of drawings of the structure of $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ is shown in Fig. 1.

The discrepancy factor $R=\Sigma| | F_{o}\left|-\left|F_{c}\right|\right| / \Sigma\left|F_{o}\right|=0.0536$ for 489 independent reflections. The standard deviation of an observation of unit weight, $\left[\Sigma w\left(F_{o}-F_{c}\right)^{2} /\left(n_{o}-n_{v}\right)\right]^{1 / 2}$ where $n_{o}$ is the number of reflections and $n_{v}$ the number of variables, is $1 \cdot 307$. The interatomic distances are listed in Table 2, and the observed and calculated structure factors are listed in Table 3.

Table 2. Interatomic distances $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$

| $\mathrm{Zr}-\mathrm{F}(4)$ | $2 \cdot 012$ (7) $\AA$ | $\mathrm{Na}(1)-\mathrm{F}(3)$ | $2 \cdot 218$ (9) $\AA$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{Zr}-\ldots-\mathrm{F}(3)$ | $2 \cdot 019$ (8) | $\mathrm{Na}(1)-\mathrm{F}(1)$ | $2 \cdot 249$ (9) |
| $\mathrm{Zr}--\mathrm{F}(2)$ | $2 \cdot 024$ (7) | $\mathrm{Na}(1)-\mathrm{F}(4)$ | $2 \cdot 264$ (9) |
| $\mathrm{Zr}-\mathrm{F}(5)$ | 2.035 (7) | $\mathrm{Na}(1)-\mathrm{F}(3)$ | $2 \cdot 35$ (1) |
| $\mathrm{Zr}--\mathrm{F}(1)$ | 2.058 (8) | $\mathrm{Na}(1)-\mathrm{F}(2)$ | 2.41 (1) |
| $\mathrm{Zr}--\mathrm{F}(6)$ | $2 \cdot 138$ (7) | $\mathrm{Na}(1)-\mathrm{F}(6)$ | $2 \cdot 53$ (2) |
| $\mathrm{Zr}-\mathrm{F}(6)$ | $2 \cdot 167$ (6) | $\mathrm{Na}(1)-\mathrm{F}(5)$ | $2 \cdot 62$ (2) |
| $\mathrm{Na}(2)-\mathrm{F}(5)$ | $2 \cdot 211$ (8) | $\mathrm{Na}(2)-\mathrm{F}(1)$ | 2.429 (9) |
| $\mathrm{Na}(2)-\mathrm{F}(1)$ | $2 \cdot 302$ (8) | $\mathrm{Na}(2)-\mathrm{F}(4)$ | 2.447 (9) |
| $\mathrm{Na}(2)-\mathrm{F}(5)$ | $2 \cdot 33$ (1) | $\mathrm{Na}(2)-\mathrm{F}(2)$ | $2 \cdot 66$ (1) |
| $\mathrm{Na}(2)-\mathrm{F}(2)$ | $2 \cdot 369$ (9) | $\mathrm{Na}(2)-\mathrm{F}(4)$ | $2 \cdot 922$ (8) |
| $F(1)--F(5)$ | 2.515 (9) | $F(1)-F(6)$ | $2 \cdot 54$ (1) |
| $F(1)-F(2)$ | $2 \cdot 54$ (2) | $F(1)-F(2)$ | $2 \cdot 89$ (2) |
| $F(1)-F(2)$ | $2 \cdot 98$ (2) | $F(2)-F(5)$ | 2.595 (9) |
| $2[F(2)-F(6)]$ | $2 \cdot 90$ (1) | $F(3)-F(5)$ | 2.569 (9) |
| $F(3)-F(6)$ | $2 \cdot 685$ (9) | $F(3)-F(6)$ | 2.74 (2) |
| $F(3)-F(6)$ | $3 \cdot 000$ (9) | $F(4)-F(5)$ | $2 \cdot 52$ (2) |
| $F(4)-\mathrm{F}(6)$ | $2 \cdot 665$ (9) | $F(4)-F(5)$ | $2 \cdot 968$ (8) |



Fig. 1. The crystal structure of $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$. The four nearest neighbor $\mathrm{Na}(1)$ and $\mathrm{Na}(2)$ polyhedra are shown around one Zr polyhedron. One fourth unit-cell outlined.

Table 3. Observed and calculated structure factors for $\gamma-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$


The $\mathrm{Zr}^{4+}$ ion and the $\mathrm{Na}(1)^{+}$ion are surrounded by an irregular array of 7 F - ions. The resulting $\mathrm{Zr}-\mathrm{F}$ polyhedron has nine triangular faces and the $\mathrm{Na}(1)-\mathrm{F}$ polyhedron has
three triangular faces and two trapezium faces. Zachariasen (1948) found similar $\mathrm{Zr}-7 \mathrm{~F}$ polyhedra in the structure of $\mathrm{Na}_{3} \mathrm{ZrF}_{7}$. The $\mathrm{Zr}-\mathrm{F}$ polyhedron shares edges with three $\mathrm{Na}(1)-\mathrm{F}$ polyhedra and a corner with one other. The $\mathrm{Na}(2)^{+}$ ion is coordinated by $8 \mathrm{~F}^{-}$at the corners of an six faced trapezohedron. The $\mathrm{Zr}-\mathrm{F}$ polyhedron shares edges with four of the $\mathrm{Na}(2)-\mathrm{F}$ trapezohedra. There are two other $\mathrm{F}^{-}$ polyhedra similar to $\mathrm{Na}(1)$ and $\mathrm{Na}(2)$ with centers at approximately $x=0.25, y=0.04, z=0.75$ and $x=0.33, y=$ $0 \cdot 50, z=0.90$. These polyhedra are vacant and too small to contain Na or Zr cations. The final difference electron density map has no peaks greater than $1 \cdot 60$ e. $\AA^{-3}$ and scattering matter put on the vacant sites does not give a model which converges with a least-squares refinement. The vacancies perhaps explain why this polymorph is metastable with respect to $\Delta-\mathrm{Na}_{2} \mathrm{ZrF}_{6}$ below $460^{\circ} \mathrm{C}$.

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Indexing of the $\psi$-sulfur fiber pattern. By S. Geller and M. D. Lind, Science Center, North American Rockwell Corporation, Thousand Oaks, California, U.S.A.
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#### Abstract

The fiber pattern of $\psi$-sulfur reported by Tuinstra and the rotation photograph of the pressure-induced fibrous modification of sulfur (II) about the fiber axis (a) are essentially the same and have been indexed completely on the pseudo-orthorhombic $C$-face-centered cell with $a=13 \cdot 8, b=32 \cdot 4$ and $c=9 \cdot 25 \AA$.


We have recently completed a study of the structure of the pressure-induced fibrous form of sulfur (Lind \& Geller, 1969). There is strong evidence that this form of sulfur is the same as the $\psi$-sulfur reported by Prins, Schenk \& Wachters (1957; see also Prins \& Tuinstra, 1963). Especially important is the exact match of the rotation photograph about the fiber (a) axis of a crystal of the pressure-induced phase and that of a fiber pattern of the $\psi$-sulfur.* Inasmuch as the literature (Tuinstra, 1966, 1967) contains questionable conclusions regarding the indexing of this pattern, it seemed worthwhile to give the results which follow.
It has already been reported (Geller, 1966) that the single-crystal-type diffraction data from the pressure-induced phase indicated that the crystals are $C$-centered orthorhombic with lattice constants $a=13 \cdot 8, b=32 \cdot 4$ and $c=$ $9 \cdot 25 \AA$. The structure determination (Lind \& Geller, 1969) has led to the conclusion that the crystal symmetry is more likely $P 2$ and that the apparent orthorhombic symmetry results from a fine-grained twinning. The true monoclinic cell then has the lattice constants $a=17 \cdot 6, b=9 \cdot 25, c=$

[^1]$13.8 \AA, \beta=113^{\circ}$. The orthorhombic indices listed for the powder pattern (Geller, 1960) may be transformed to the monoclinic indices by application of the two matrices $\frac{\overline{1} 11}{2} 0|001| 100$ and $\frac{1}{2} \frac{1}{2} 0|001| \overline{100}$ to each reflection.

We show the indexing of the rotation photograph in Table 1. Listed in the first column are Tuinstra's (1966) observed values, $Q_{0}\left(Q=10^{4} / d^{2}\right)$, measured on his fiber photographs of the stretched, $\mathrm{CS}_{2}$-extracted, annealed fibrous sulfur. In the second column, we give our values of $Q_{0}$, measured on a rotation photograph ( 2 hr exposure, 57.3 mm dia. camera, $\mathrm{Cu} K \alpha$ radiation, Ni filtered) taken of the same crystal used to obtain the data in the paper by Lind \& Geller (1969). (The photograph to which Tuinstra (1967) refers is exactly the same except perhaps for exposure time.) We do not list the qualitative intensities; as we said earlier, the photographs of stretched, $\mathrm{CS}_{2^{-}}$ extracted, annealed fibrous sulfur and pressure-induced fibrous sulfur superimpose exactly and quantitative $F_{n k l}$ are given in the Lind \& Geller (1969) paper. We see that the two sets of $Q_{o}$ agree quite well although ours are considerably better resolved. Our $Q_{c}$ and indices based on the pseudo-orthorhombic lattice constants are given in the third and fourth columns, respectively. It is seen that the agreement in $Q$ 's is excellent, so that even though it is possible that the fiber axis is very long, as Tuinstra (1966) sug-


[^0]:    * Research sponsored by the U.S. Atomic Energy Commission under contract with Union Carbide Corporation.

[^1]:    * The best $\psi$-sulfur photograph we have seen has been made by J.Donohue and S.H. Goodman. This is the one that superposes exactly on our (pseudo-orthorhombic) $a$-axis rotation photograph.

